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Construction of Reality and Its Influence on the Understanding of Quantum Structures

Diederik Aerts[1]

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We present a view where the process of detection of existing reality and creation of new reality is considered explicitly, and that rests on the insight that "*the present*" is operationally connected to the collection of the "*possible*" experiences that we can live. We developed a formalism in this view and reality appears as a subtle construction from the reservoir of possible experiences, and see that our consciousness of this construction is lost while many deep problems are related to its presence. We apply the formalism to explain the non-classical probability structures in quantum physics and outline the appearance of topological structures of "*the present.*" This structuring process is going on actually giving rise to the flow of reality.

We present a formalism to analyze "the construction of reality." We want to use this formalism to understand some aspects of the reality of the classical and the quantum worlds on a deeper level. A more complete presentation of the formalism also including applications to other fields than classical and quantum mechanics can be found in Aerts (1990*c*).

One of the aims is to detect hidden prejudices that are unconsciously used to interpret and criticize certain parts of reality that do perhaps not obey these prejudices. It will follow that some of the difficulties that we have in understanding quantum reality are due to prescientific prejudices about the way in which we think that reality *has* to be, prejudices which we have completely forgotten about. We will try to put in evidence some of these prescientific prejudices. To do this we will have to introduce new concepts. We will see, however, that the concepts needed to understand the aspect of the quantum reality that we want to investigate in this paper are not mysterious and un-understandable. They exist and are used frequently in situations

[1]Department of Theoretical Physics, Vrije Universiteit Brussel, 1050, Brussels.

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of everyday life. Therefore we will introduce these concepts by means of examples of everyday life, such that they are intuitively clear.

The formalism is not yet completed; many steps are unfinished and have to be investigated further. But the methodology is clearly stated. In this sense it can be a starting point for further research on the understanding of other parts of human reality.[2]

I started to work on the elaboration of this formalism in the early eighties. Several works have inspired me. First of all there is the research on quantum physics and the elaboration of the state-property theory in Geneva under the guidance of Constantin Piron, in which I have been collaborating. This state-property theory delivers a realistic interpretation of ordinary quantum mechanics, but as a theory goes far beyond it (Piron, 1976, 1989, 1990; Aerts, 1981, 1982, 1983). The formalism presented here and in Aerts (1990*c*) can in a certain sense be used to found the state-property theory (Piron, 1976, 1989, 1990; Aerts, 1981, 1982, 1983), a relation that is explained in detail in Aerts (1990*c*). There is the work of Charles Randall and David Foulis, their (still unended but fascinating) quest for the construction of a universal language for the sciences (Randall and Foulis, 1983*a*,*b*; Foulis and Randall, 1985). Also their work has been of great value for the elaboration of certain aspects of the presented formalism. The relation is explained in Aerts (1990*c*). There are the many works of those doing research in and around the field commonly called quantum logic.[3] There is also the work and thoughts of Henri Poincaré (1902), which for the first time made me understand that it is not *a priori* necessary to consider the world as given, and we observing it. Poincaré analyzed already (before the existence of quantum mechanics) the subtle role of man in the construction of some aspect of reality, such as Euclidean space. There is the work of Jean Piaget, which has convinced me that a lot of this prescientific construction of reality happens again in the early years of childhood for every human being (Piaget, 1969; Piaget and Inhelder, 1971). There is the book by Erwin Schrödinger (1944), which made me understand how to approach the problem of life in the formalism, and made me find the energy to go and study the findings of the neo-Darwinists. And there is also the more recent and fruitful collaboration with Jean Reignier, on the quantum problematic of nonlocality and its

[2]We can think of the social, economic, and cultural parts of human reality, and of the psychological, medical, biological, and biochemical aspects of ourselves (human body and human mind), and the connections that have to be made with theories studying these domains.

[3]References on quantum logic can be found in the proceedings of two conferences (Beltrametti and van Fraassen, 1981; Mittelstaedt *et al.*, 1985) and also in the excellent survey book by Beltrametti and Cassinelli (1981). The collection of researchers working in the domain called "quantum logic" is in fact a collection of people studying "quantum structures" in general. Historically this domain has been called quantum logic, although the research on the purely logical aspects of the quantum structure is only one aspect of the content of the domain.

connection with the construction of reality, in which we propose an operational definition for the concept of quantum nonlocality, and are investigating its relation to the construction of macroscopic space Aerts and Reignier (1991*a*,*b*).

Actually the formalism, of which only a little part is presented in this paper, is still in full development, and it is my opinion that a lot of other scientific problems can be expressed in it, with the aim of bringing some clarification among the problems and pseudoparadoxes, and in this sense coming closer to the understanding of the real mystery of our existence.

We shall begin by describing how we as human beings come into contact with the world and start constructing reality.

1. THE LIVING OF AN EXPERIENCE

When we are born, there is not much reality in our world. The reality of the others, the older people, is already around us, but we cannot understand it, neither can we influence it. But our self is already constructed in this way that we constantly interact with it. Our world is a stream of such chaotic interactions. It is from this stream of interactions that we start constructing our personal reality. We do not make this construction as an isolated human being. Constantly we are in contact with other human beings, also still further constructing their realities, and with animals also constructing their realities, and with plants, and with material objects, and with . . . .

We live certain parts of this stream of interactions. These parts we will call *experiences*. When we live such an experience, we will say that this experience is *present*, and we will call it our *present experience*.

The experience that we live is present and we will call it our present experience.

2. THE IDENTIFICATION OF AN EXPERIENCE

To be able to use an experience for the construction of our reality, we have to be able to identify it. The first time I tasted sweetness was when I got honey on my comforter. This was the living of a new experience. The collection of family pictures shows that this experience was repeated often afterward. In this way this experience got an identity. I could identify "The tasting of sweetness," which I will call experience $\mathbf{E_1}$**(I taste sweetness)**. This experience of tasting sweetness happened to me. I could not control it, nor could I predict it. But I could identify it. And I liked it, this (as I know now) because by means of the cells of my tongue, certain parts of my brain were triggered. And the triggering of these parts gave me a feeling of satisfaction.

Therefore it would be nice for me to be able to control more or less the happening of this experience, such that I could look it up. After some time it became clear to me that when I cried hard enough, sometimes (certainly not always) the nice experience of tasting sweetness happened to me. This "tasting sweetness" experience seemed to be a particular type of a more general experience.

I could taste also other things than just sweetness. Some of it (food?) tasted good, and some of it tasted bad. But clearly I could identify the experience of tasting, which I will denote $\mathbf{E_2(I\ taste)}$.

Of course millions of other experiences happened to me in my first life-months. I could touch things. Let me denote this experience $\mathbf{E_3(I\ touch)}$. This was a very interesting experience. I also could identify it. And amazingly enough, compared to the tasting that more or less just happened to me, this touching could be controlled much better. If I wanted to I could start touching, and if I wanted to I also could stop touching.

I could see things. This was even a more amazing experience with a lot of subtle variations in it, but anyhow easy to identify. Again I could control more or less this experience. I could stop seeing, and I could start seeing. But once I had decided to start seeing, not much control over what I was seeing was left over to me. Let me denote this experience by $\mathbf{E_4(I\ see)}$.

I could hear things. Again a very identifiable experience. It was less controllable than the seeing and the touching. It more or less also happened to me, and in this sense resembled more the tasting experience. The difference was that the hearing was almost always there, while the tasting experience only happened from time to time. Let me denote this experience by $\mathbf{E_5(I\ hear)}$.

3. THE POWER OVER AN EXPERIENCE

If an experience remains that unreachable for me such that the only thing I can do with it is to live it and identify it, it will not contribute very much to the construction of my reality. The reason is very simple. Such an experience only happens to me in an unpredictable and uncontrollable way, and although I can live it and identify it when it happens to me, that is the only thing I can do with it. Such an experience exists for me as a possible experience. But that is all. Therefore such an experience I will call an **unreachable experience**.

All of us still have these kinds of experiences in our reality, but most of us do not pay much attention to them, and since they do not contribute to the construction of our reality, they are not very important for us. But sometimes a human being can come in a situation where such unreachable experiences happen in such a way that they become important experiences.

The more this happens, the more a human being will start living in a miserable and unreachable reality. And sometimes this human being will more *be lived by* than *live*.

The first step for the construction of our reality starts with the gathering of power over our living of the experiences. This power can have very different forms. We will analyze them in the next sections.

4. THE POWER OF KNOWLEDGE AND THE POWER OF CREATION

For those experiences that just happen to me when I live them, such as the tasting experience and the hearing experience, there is one way of getting power over them. That is by trying to know them. Let us give an easy example to show what we mean.

I can hear my mother's heartbeat. And even if this hearing just happens to me, I know that after one heartbeat, another always will follow. This knowledge gives me a power over the experience, in the sense that it does not just happen to me, because I know it will happen, and hence I can predict it. Let me denote this experience of hearing my mother's heartbeat by $\mathbf{E_6}$**(I hear my mother's heartbeat)**. So from the class of all possible experiences those experiences that can be known will be a special subclass.

To know an experience is to have the power to ***predict*** *all the aspects of this experience while I live it.*

Even if the heartbeats just happen to me, I can more or less predict when one of these heartbeat experiences will come. Later I learn to have the power of prediction about aspects of much more complicated experiences. Let us observe that knowledge is a passive power. On the other extreme we have the active power of creation.

If I start crying to evoke the experience of tasting sweetness, then I am trying to get another kind of power than the power of knowledge. I have used my power of creation. It is with this same power of creation that I can control the experience of touching and that I can close my eyes if I want to stop the experience of seeing.

To create an experience is to have the power to ***control*** *all the aspects of this experience while I live it.*

There exists no experience that we know completely, nor there exists an experience that we create completely. For all experiences we try both types of power, and usually we alternate between creation and knowledge if we really want to have power over an experience. It is by means of this approach

that we start using our experiences to construct our reality. How we do this will be the investigation of the following sections.

Human beings are entities that have these two powers, the power of knowledge and the power of creation. And it is by using these two powers that we construct our reality.[4]

5. MY PERSONAL TIME

The most primitive order that can be introduced is the order of before, present, after. The experience that I live is present. The experiences that I have lived are past, and the experiences that I will live are future. In this sense in principle I could start counting the present experiences one after the other. The series of numbers that I get in this way is the root of my personal time. Since this time is personal, the only properties I attribute to it are subjective properties. It never stops and always flows forward. It can change in speed. Sometimes it goes fast, and sometimes it slows down. It is carried by those aspects of my experiences that I do not create or control and cannot predict. The aspect that makes that "I have to experience."

Personal time is the most primitive construction of my reality, the construction that could already have been made by means of only **unreachable** experiences. Indeed, even if all my experiences would be unreachable, hence I can live them, and identify them, then personal time would already have been constructed. Therefore time gives us this feeling of not being able to escape to it, exactly because in it we have structured the unpredictable, uncontrollable, and hence unreachable aspects of our experiences.

6. MY FIRST STEPS OUT OF PERSONAL TIME

With those experiences that are not unreachable, a lot more of reality can be constructed. Indeed, if I have power of knowledge and power of creation over an experience, this means that I have chosen more or less to

[4]Some ages, and some cultures, have given more attention to one of the two powers, and depending on this, the reality constructed by such a culture will be different. When more attention is given to the power of "knowledge," the constructed reality will be more exterior. Humans will be seen as a little part of the huge "universe of knowledge" and in the extreme of this attitude a fatalistic reality will be constructed, a reality where everything is controlled and created by the external fatum. When more attention is given to the power of "creation," the constructed reality will be more individualistic. Humans will be seen as the masterpiece of the universe, and in the extreme of this attitude an egoistic reality will be constructed, a reality where everything is controlled and created by the self. During the ages of existence of humanity, the constructed reality always has fluctuated between these two extremes, and it is easy to find examples in the history of civilization. It is not our aim to study the social aspects of these fluctuations, although they are very deep and important, but we want to study the effects of these fluctuations on the primitive and in general prescientific construction of reality.

live this experience. The living of the particular experience is not imposed upon me. I have **chosen** to live it. But from this fact follows that I could have made another choice, and I could have lived another experience **in replacement** of the present experience. It is very important to understand the consequences of this fact. Since I could have chosen to live another experience in replacement of the present experience, it makes sense to introduce the concept of **possible experience**. I give a concrete example to explain what I mean.

I consider the following situation: I am inside my house in Brussels. It is night, the windows are shut. I sit in a chair, reading a novel. I have a basket filled with walnuts at my side, and from time to time I take one of them, crack it and eat it. My son is in bed and already asleep. New York exists and is busy.

My present experience is E_7**(I read a novel)**. But, a lot of other things happen while I am living this present experience. These things happen in my present reality. While "I am reading the novel" some of the happenings that happen are the following: H_7**(the novel exists)**, H_8**(the inside of my house in Brussels exists)**, H_9**(it is night)**, H_{10}**(the basket and the walnuts exist, and are at my side)**, H_{11}**(my son is in bed and is sleeping)**, H_{12}**(New York exists and is busy)**. All these happenings, and much more, happen while I live the present experience E_7**(I read a novel)**.

Why have I constructed reality in such a way that what I am just saying is evident for everybody (and therefore shows that we are not conscious of the construction that is behind this evidence)?

Certainly it is not because I experience also these other happenings. My only **present** experience is the experience of reading the novel. But, and this is the reason for this type of construction, I could have used my power to live an experience including one of the other happenings **in replacement** of my present experience. Let me put down the list of these experiences that I could have chosen to experience in replacement of my present experience: E_8**(I observe that I am inside my house in Brussels)**, E_9**(I see that it is night)**, E_{10}**(I take a walnut, crack it, and eat it)**, E_{11}**(I go and look in the bedroom to see if my son is asleep)**, E_{12}**(I go to New York, and observe that it is busy)**.

This example indicates how we have started to construct reality. First of all we have tried to identify the two main aspects of every experience. The aspect that is controlled and created by me, and the aspect that just happens to me and can only be known by me. Let us introduce this distinction in a formal way.

7. CREATIONS AND HAPPENINGS

To see what I mean, let us consider the experience E_{10}**(I take a walnut from the basket, crack it, and eat it)**. In this experience, there is an aspect

that is an action of me, the taking and the cracking, and the eating. There is also an aspect that is an observation of me, the walnut and the basket. By studying how our senses work, I can indeed say that it is the light reflected on the walnut, and on the basket, that gives me the experience of walnut and the experience of basket. This is an explanation that only now can be given; it is, however, not what was known in earlier days when the first world-models of humanity were constructed. But without knowing the explanation delivered now by a detailed analysis, we could see very easily that an experience contains always these two aspects, a **creation**-aspect, and an **observation**-aspect, simply because our will can only control part of the experience. This is the creation-aspect.

For example, in E_7**(I read a novel)** the reading is created by me, but the novel is not created by me. In general we can indicate for an experience the aspect that is created by me and the aspect not created by me. The aspect not created by me lends itself to my creation. We can reformulate an experience in the following way: E_{10}**(I take a walnut, crack it, and eat it)** becomes E_{10}**(A walnut is taken by me, and lends itself to my cracking and eating)** and E_7**(I read a novel)** becomes E_7**(The novel lends itself to my reading)** . . . .

The taking, cracking, eating, and reading will be called **creations** or actions and will be denoted by C_{10}**(I take, crack, and eat)** and C_7**(I read)**. The walnut and the novel will be called **happenings** and will be denoted by H_{10}**(the walnut)** and H_7**(the novel)**.

*A **creation** is that aspect of an experience created, controlled, and acted by me, and a **happening** is that aspect of an experience lending itself to my creation, control, and action.*

An experience is determined by a description of the creation and a description of the happening. Both creation and happening can be expressed by verbs. To take, to crack, to eat, and to read, are the verbs that describe my creations in the examples. The walnut and the novel are happenings that have the additional property of being objects, which means happenings with a great stability. They are expressed by the verbs to be a walnut, to be a novel.

Every one of my experiences E consists of one of my creations C and one of my happenings H, so we can write $E=(C, H)$.

A beautiful image that can be used as a metaphor for our model of the world is the image of the skier. A skier skis downhill. At every instant he or she has to be in complete harmony with the form of the mountain underneath. The form of the mountain represents the happening. The way of being in harmony with this form represents the skier's creation. The skier's

creation, in harmony fused with the skier's happening, is his or her experience.

8. THE CONSTRUCTION OF REALITY, PRESENT, PAST, AND FUTURE

Let us again consider the collection of experiences: E_7**(I read a novel)**, E_8**(I observe that I am inside my house in Brussels)**, E_9**(I see that it is night)**, E_{10}**(I take a walnut, crack it, and eat it)**, E_{11}**(I go and look in the bedroom to see if my son is asleep)**, and E_{12}**(I go to New York, and observe that it is busy)**. Let us represent the construction of the reality that is made out of this little collection of experiences by means of the drawing represented in Figure 1.

E_7**(I read a novel)** is my present experience. In my past I could, however, at several moments have chosen to do something else and this choice would have led me to have another present experience than E_7**(I read a novel)**. For example:

At (5) I could have decided to stop reading and observe that I am inside the house. Then E_8**(I observe that I am inside my house in Brussels)** would have been my present experience.

At (4) I could have decided to stop reading and open the windows and see that it is night. Then E_9**(I see that it is night)** would have been my present experience.

At (3) I could have decided to stop reading, take a walnut from the basket, crack it, and eat it. Then E_{10}**(I take a walnut, crack it, and eat it)** would have been my present experience.

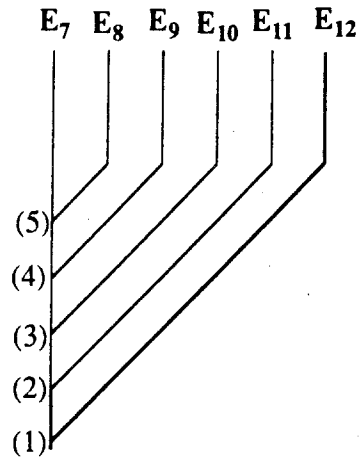

Fig. 1. A drawing representing the construction of reality.

At (2) I could have decided to go and see in the bedroom whether my son is asleep. Then E_{11}**(I go and look in the bedroom to see if my son is asleep)** would have been my present experience.

At (1) I could have decided to take a plane and fly to New York and see how busy it was. Then E_{12}**(I go to New York, and observe that it is busy)** would have been my present experience.

Even when they are not the happening aspect of my present experience, happenings "happen" at present if they are the happening aspect of an experience I could have lived in replacement of my present experience, if I would have decided so in my past.

The fact that a certain experience **E** consisting of a creation **C** and a happening **H** is for me a possible present experience depends on two factors:

(1) I have to be able to perform the creation.

(2) The happening has to be available.

For example, the experience E_8**(I observe that I am inside my house in Brussels)** is a possible experience for me, if:

(1) I can perform the creation that consists of observing the inside of my house in Brussels. In other words, if this creation is in my personal power.

(2) The happening "the inside of my house in Brussels" has to be available for me. In other words, this happening has to be contained in my personal reality.

(1) *The collection of all creations that I can perform at present I will call my present personal power.*

(2) *The collection of all happenings that are available to me at present I will call my present personal reality.*

I define as my present personal reality the collection of these happenings, the collection of the happenings that are available to one of my creations if I would have used my personal power in such a way that at present I fuse one of these creations with one of these happenings.

My present personal reality consists of all the happenings that are available to me at present. My past reality consists of all the happenings that were available to me in the past. My future reality consists of all the happenings that shall be available to me in the future.

My present personal power consists of all the creations that I can perform at present. My past personal power consists of all the creations that I could perform in the past. My future personal power consists of all the creations I shall be able to perform in the future.

Happenings can happen at once, because to happen, a happening does not have to be part of my present experience. It is sufficient that it is available, and things can be available at once. Therefore, although my present experience is only one, my present personal reality consists of thousands of happenings all happening at once. Again I want to state clearly that if I say, "At present, happening **H** happens," this does only mean that at present I could have had an experience $\mathbf{E}=(\mathbf{C}, \mathbf{H})$, where **C** is the creation I would have chosen to make in this case.

This concept of reality is not clearly understood in actual physical theories. Physical theories know how to treat past, present, and future. But reality is a construction about the possible. It is constructed by means of the experiences **I could have** lived, but probably never will live.[5]

[5]Therefore, all languages, which are in fact prescientific models of the world, contain different modes of description of this world. The indicative mode is about the happening-part of my experiences, present, past, or future. The demanding mode is about the creation part of my experiences, present, past, or future. The conditional mode is about the construction of reality. In a language I can say, being in Brussels, "New York exists at this moment," because "I could have been in New York, if I would have decided so in the past." Try to say the same without using the conditional mode.

The positivist (and strict operationalist) attitude of the beginning of this century has had a negative influence on the understanding of the nature of the construction of reality. This attitude has had its great merit in helping science to get rid of many disturbing metaphysical beliefs, but at the same time has created the thought that science should limit itself to the "observable" and should not argue about the "possible." The conditional mode was considered unscientific. For example, the young Einstein was a strong adept of this attitude, and it is certainly partly this attitude that was at the origin of his analysis of the concept of simultaneity and the consequences of this analysis for the construction of relativity theory. Later Einstein changed rather fundamentally. The famous paper containing the EPR paradox (Einstein *et al.*, 1935) shows an attitude much closer to the one that we expose in this paper [what is called an element of reality by Einstein is directly related to what is called a property in Piron (1976) and what is called a happening in this paper and Aerts (1990*c*)].

One might think that the formalism presented here is realistic (philosophically speaking), and hence can only be accepted by those people who call themselves realists. I do not agree with this. In a certain sense the formalism avoids the philosophical debate around realism. Let me explain why. A new concept is introduced. It is the concept of "to happen," and it is stated that a happening can "happen" without me living the experience that contains this happening. This is a precise way of saying that I am not the creator of the happening, and that a happening can "happen" without me fusing it with one of my creations. This explains that more than one happening can happen at present. But I say more. I make explicit what are the happenings that can happen at present. The happenings that can happen at present are those that could have been fused with one of my creations if I had somewhere in the past made another choice such that my present creation would have been different than the present one. Hence in this way one defines operationally the meaning of the concept "to happen." My opinion is that the methodology of operationalism is alright, but the philosophy going with it is too limited in a lot of ways. For example, it does not seem to want to take into account the fact

The reality of the skier is the form of the complete mountain. These are the happenings waiting for the skier if eventually the skier's life passes that spot of reality. The skier's present reality is the form the mountain has in the present.

9. CLASSICAL REALITY, QUANTUM REALITY, DETERMINISM, AND PROBABILITY

Most of the modern human part of our reality we have been investigating by the act (creation) of observation. That is the reason why the constructions of modern technology are often considered to be alien to natural reality (they are considered to be artificial). This creation of observation is a very passive one, and almost only leads to a gathering of power of knowledge. This period dominated by the observational paradigm has been very long (perhaps millions of years, from the time that all our cells came together to form a body with eyes to observe), and is still dominating the nature of our actual human world-model. This situation has paved the way for the classical mechanical world model, a model where the world is considered to be a huge clockwork, and we humans observe it and gain knowledge about it, and we are ourselves part of it, and controlled by it. This classical mechanical world-model still determines a lot of the thinking in different disciplines of science. It is a world image where it is thought that the only active part of humanity in the construction of reality is an act of observation.[6]

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that constantly new objects are created by humanity, and these are to be compared as to their status of reality with existing objects. The discussion about what is real and what is not real should become a quantitative discussion. Instead of asking about the reality of a certain object, one should compare the object to others: "Is this object 'as real as' that one, or 'less real than' that one?" We shall probably in the future have to start quantifying reality. Karl Popper, by introducing his different worlds, has made an attempt in this direction. But it remains a qualitative classification, too much dominated by our actual macroscopic reality.

[6]It is well known that this is not true, and probably the great social, economic, and cultural crisis that humanity is living at this moment is due to the fact that the observational paradigm is changing. Humans shall have to start a construction of reality (of human society, hence its economic, social, and cultural aspects) that will be again more of the creation type (as had to be done during the period where all our cells came together to form a body). But a lot of resistance against such an international creation-like enterprise exists for a multitude of reasons. I think that one of the fundamental reasons for the resistance to such a project is the fact that we as humans have not yet freed ourselves of the observational paradigm. It is time that humanity starts to get conscious of this fact, and takes responsibility for its creations, by planning them, and taking into account its effects on the construction of long-term future reality. There will be no choice (since these periods are determined by profound long-term laws going together with the methodology of humanity to construct reality) if humanity wants to avoid the risk of collapsing into a chaotic reality with the only possibility of living "unreachable" experiences.

This of course is obviously wrong, and in research on the nature of quantum reality, physicists have been forced to suffer the consequences of this wrong set of ideas. The fact that this classical set of ideas is wrong is so obvious that it has been very difficult to recognize. The consequences of this wrong set of ideas have shown up inside the very difficult formalism of quantum mechanics, where anyhow nobody really knew what were the creations and what were the happenings. This fact is the main reason for the existence of so many paradoxical aspects related to quantum mechanics and the reality of the quantum world.

Classical reality, as it appears to us from the classical physical theories, is deterministic. There exists a probability theory describing situations inside this classical reality, but the lack of determinism in these probabilistic situations is explained by interpreting the probability as describing the lack of knowledge that we have about the complete reality of the situation. It is a probability theory conditioned by the observational paradigm of the classical world-view. As we shall see, now that we have introduced the necessary elements to discuss clearly the concept of reality itself, the state of affairs is not that simple. Probability can show up very naturally in another way, and then give rise to quantum mechanical probability structures.

We consider the same situation. I am inside my house in Brussels, and I am reading a novel. But we will concentrate a little bit more on the experience formed by the creation $\mathbf{C_{10}}$**(I take a walnut, and crack it, with the aim of eating it)**. I must explain a little bit more about the actual creation that I am performing. I do not use a professional tool for cracking the walnut. I just take it between my hands, put all the force I have, and see what comes out. Everybody who has had some experience with this way of eating walnuts knows that with a given walnut as happening $\mathbf{H_{10}}$**(a walnut)**, different experiences can follow. Let us analyze some of the possibilities.

1. *If a walnut after cracking turns out to be moldy, then I do not eat it.*

This means that for a particular walnut number k (let us call this particular walnut happening $\mathbf{H}_{10,k}$ of the basket containing N walnuts), my creation $\mathbf{C_{10}}$ can fuse with this happening and give rise to different experiences. Let us denote two types of them by $\mathbf{E}_{10,1}$**(I crack the walnut, and eat it)** and $\mathbf{E}_{10,2}$**(I crack the walnut, and do not eat it)**.

Suppose that, of the N walnuts in the basket, M are moldy. Then the probability that walnut k will lead to experience $\mathbf{E}_{10,1}$ is given by $(N-M)/N$, and the probability that walnut k will lead to the experience $\mathbf{E}_{10,2}$ is given by M/N. This is a probability that finds its origin in my lack of knowledge of the complete reality of the walnuts. Indeed, everybody will believe me when I suppose that already before I start to perform my creation of cracking the walnut number k, it was moldy or not, and if I would have been able to

know this, I could have eliminated the probability by only taking those walnuts that were not moldy, and only trying to eat these ones. The whole of classical probability theory is based on this way of thinking about reality.

Everybody who has some experience with cracking walnuts knows that other things can go wrong such that it will not be possible to eat the walnut. Indeed, sometimes the cracking destroys the walnut in such a way that the nut is completely mixed up with the shell. If this happens, I will have a quick investigation of whether it is still possible to disentangle them, and if it is not the case, I will not eat the walnut.

2. *If a walnut, after being cracked, is too mixed up with the shell, then I do not eat it.*

This again leads to the fact that for a particular walnut number k, hence happening $\mathbf{H_{10,k}}$, the fusing with my creation $\mathbf{C_{10}}$ can give rise to two different experiences. Experience $\mathbf{E_{10,3}}$**(I crack the walnut, and eat it)** and $\mathbf{E_{10,4}}$**(I crack the walnut, and do not eat it)**. And again, as in the foregoing, we can attribute a probability to the two possible experiences.

We immediately feel that this type of probability is of a different nature, and depends on the way the cracking will be performed (depends on the creation aspect of the experience). Indeed, unlike the foregoing case of the M walnuts that are moldy and the $N-M$ walnuts that are not moldy, we cannot subdivide the collection of N walnuts in the basket *a priori* (before the cracking takes place) into those that I will eat because they are cracked well, and those that I will not eat because they are cracked wrongly. Such a subdivision does not exist before the creation $\mathbf{C_{10}}$ of cracking. Hence here we have an explicit example of how a part of reality is constructed by me, cracking the walnuts.

The interesting fact is now that the mathematical probability structure that is needed to describe the indeterminism (which in our formalism means that equivalent creations and equivalent happenings can fuse together to deliver different experiences) that follows from such a kind of situation is different from the mathematical probability structure needed to describe the indeterminism that follows from situations where before the creation (and hence before the experience) there is a more complete reality of the happening in question, of which we lack the knowledge. And what is even more interesting is that:

The probability structure describing indeterminism that comes from a lack of knowledge on the more complete reality of the happening in question is a classical-like probability structure.

The probability structure describing indeterminism that comes from the fact that during the creation a new piece of reality is created, not existing before the creation, is a quantum-like probability structure.

We do not have the space in this paper to prove these two important statements, but we can refer to earlier work, where (not in the context of this formalism) the statements were proven in general, and by means of concrete examples (Aerts, 1986, 1987, 1988, 1990*a*,*b*). Elsewhere (Aerts and Von Bogaert, 1992) we repeat one of the examples of Aerts (1986, 1987, 1988, 1990*a*,*b*), showing explicitly that it has a quantum logical structure, although it only consists of mechanical macroscopic physical entities. A complete and general proof inside the present formalism can be found in Aerts (1990*c*).

10. THE CRACKING OF WALNUTS AND QUANTUM REALITY

Let us now consider the situation where we have been able to put away all the moldy walnuts of the basket. So I am in the situation to have next to me a basket with only nonmoldy walnuts. In classical mechanics language one would say that every individual walnut in the basket is now in a pure state as to the property of being nonmoldy. In the original situation, where the moldy walnuts were still present, an individual walnut would have been presented in classical mechanics formalism by a mixed state of moldy, nonmoldy walnut, with weights given by M/N and $(N-M)/N$.

In this new situation we have a basket with nonmoldy walnuts. We consider now a more refined happening $\mathbf{H_{13}}$**(nonmoldy walnut)**. With the creation $\mathbf{C_{13}}$**(I crack)** it can be fused to deliver an experience $\mathbf{E_{13}}$**(I crack a nonmoldy walnut)**. However, taking into account the reasoning of the foregoing section, even such a nonmoldy walnut will not always be eaten. I will eat some of these walnuts, and will not eat others of them, depending on the quality of the cracking. The cracking of the walnut changes its state in such a way that (if the cracking succeeds) it becomes really ready to be eaten or (if the cracking does not succeed) it has become uneatable. In this sense we must introduce another concept, and say that although the basket is now full of nonmoldy walnuts, since the cracking still has to take place, they are only *potentially* ready to be eaten.

Nobody has any problem in understanding the example of the walnut. Our proposal is now that we have to use our human mind to understand quantum reality in analogy with this kind of very natural and very abundant situation in our everyday life. Of course, in the quantum reality the state of **potentiality** appears in relation with other happenings than the happening of "ready to be eaten."

Walnuts are only eatable or uneatable after they have been cracked. We could state this by saying that walnuts are potentially eatable and potentially uneatable.

Quantum entities in general are potentially localized and potentially nonlocalized in a certain region R of space. And the experience of finding or not finding the quantum entity in this region of space is taking place after real apparatuses to detect the quantum entity have been installed in the laboratory and the interaction of the quantum entity with these apparatuses has begun. Before they are potentially **present** and potentially **nonpresent** in region R.

The same can be said for the property spin of a quantum entity, and in relation with this property we have worked out in detail a macroscopic mechanistic example giving rise to the quantum mechanical probability model of the spin of a spin-1/2 quantum particle (Aerts, 1986, 1987). What we have called measurements in Aerts (1986, 1987) would be called creations in the formalism of this paper, and what we have called states of the entity in Aerts (1986, 1987) would be called happenings in the formalism of this paper. Also using this way of interpreting quantum reality we could give an example of a mechanistic laboratory situation giving rise to a violation of the Bell inequalities exactly with the same numerical values as the EPR violations (Aerts, 1990*b*). The problem of potential locality (or potential presence) of quantum entities is analyzed by means of the examples of concrete laboratory experiments in Aerts and Reignier (1991*a*,*b*).

We have to come back now to our analysis of the construction of reality to be able to understand the real nature of quantum reality. Let me consider once again the same situation: My present experience is $\mathbf{E_7}$**(I read a novel).** And while "I am reading the novel," we consider the following happenings: $\mathbf{H_7}$**(the novel exists),** $\mathbf{H_8}$**(the inside of my house in Brussels exists),** $\mathbf{H_9}$**(it is night),** $\mathbf{H_{10}}$**(the basket, and the walnuts exist, and are at my side),** $\mathbf{H_{13}}$**(each walnut of the basket is potentially eatable and potentially uneatable),** $\mathbf{H_{11}}$**(my son is in bed and is sleeping),** $\mathbf{H_{12}}$**(New York exists and is busy).**

The construction of my reality consists in accepting the existence of these happenings, because I could have decided in the past to use my free will and fuse one of my creations with one of these happenings and hence have one of the experiences. We want to consider now the happening $\mathbf{H_{13}}$**(each walnut of the basket is potentially eatable and potentially uneatable)** and the creation $\mathbf{C_{13}}$**(I crack)** that I could have fused with it to have one of the experiences $\mathbf{E_{13,1}}$**(I crack a walnut and eat it)** or $\mathbf{E_{13,2}}$**(I crack a walnut and do not eat it).**

We see that the construction remains valid. The fact that the fusing of $\mathbf{H_{13}}$**(each walnut of the basket is potentially eatable and potentially uneatable)** with $\mathbf{C_{13}}$**(I crack)** can give rise to two different experiences, $\mathbf{E_{13,1}}$**(I crack a walnut and eat it)** or $\mathbf{E_{13,2}}$**(I crack a walnut and do not eat it)** does not destroy the validity of the construction of reality as we have presented it. But "observer-minded" as we are conditioned, we indeed have the feeling that the

happening H_{13}**(each walnut of the basket is potentially eatable and potentially uneatable)** is "less real" than the other happenings that happen while I am reading the novel. This is the reason why we have also this, for so many physicists uneasy, feeling in relation with quantum entities. They seem to exist "less" than ordinary macroscopic entities. I think it is not important to discuss whether this is true or not as a matter of fact. What is important is to understand why we have this feeling, and how it can be explained and understood by means of everyday examples of experiences. And this was one of the aims of this paper.

In the next sections I shall propose an explanation for the fact that happenings such as H_{14}**(an entity is potentially localized and potentially nonlocalized in a certain region R of space)** do not exist in our macroscopic reality, while in the quantum reality they do exist. To investigate this, we have to start by introducing aspects of this macroscopic material reality.

11. THE INTRODUCTION OF MATERIAL TIME AND A TOPOLOGICAL STRUCTURE FOR REALITY

I want to proceed with the development of the formalism, and introduce aspects of the construction of reality that are already closer to our everyday classical conception of it.

From ancient times humanity has been fascinated by happenings going on in the sky, the motion of the sun, the changes of the moon, the motions of the planets and stars. These happenings in the sky are **periodic**. By means of these periodic experiences, humans started to **coordinate** the other experiences. They introduced the counting of the years, the months, and the days. Later on watches were invented to be able to coordinate experiences of the same day. And in this sense **material** time was introduced in the reality of the human species. Since reality is a construction, also material time is part of this reality construction. Indeed, my present experience very seldom is a material timelike experience. But in replacement of my present experience, I always could have consulted my watch, and in this way have a material time experience E_{14}**(I consult my watch)**. So, although my present experience is very seldom a material time experience, my present reality always contains a material time happening, namely the happening H_{14}**(my watch)**, which is the happening that with the creation C_{14}**(I consult)** forms the experience E_{14}**(I consult my watch)**. By means of this material time happening I coordinate my present reality.

By means of material time I coordinate my reality.[7] By means of time I can also attribute a magnitude or length to a particular experience. To do

[7] I use the term "material" time because indeed all watches used nowadays by mankind are based on periodic happenings of material objects.

this I introduce a unit particle of time. Let me call it the chronon. This can be done by referring to a well-known periodic experience. Each experience can then be measured by the number of chronons it takes from the beginning till the end of the experience. A reality that contains material time happenings has a certain metrical structure because by means of time I can give a more precise definition of my present reality.

My present reality is the collection of the happenings of all the experiences I could have lived at the same moment of time as my present experience.

Let us look again at Figure 1. By measuring the time needed to go from (5) to $\mathbf{E}_7$ and from (5) to $\mathbf{E}_8$, I can give a measure of the distance between experience $\mathbf{E}_7$ and experience $\mathbf{E}_8$. To go from "reading the novel" toward "observing that I am in the room" will take me very little time. Let us say 1 chronon (this depends on how we define the chronon). Then we put $d(\mathbf{E}_7, \mathbf{E}_8) = 1$ chronon. To check whether it is night takes me 10 chronons. Thus we will put $d(\mathbf{E}_7, \mathbf{E}_9) = 10$ chronons. To take a walnut, crack it, and eat it takes me 50 chronons. Hence $d(\mathbf{E}_7, \mathbf{E}_{10}) = 50$ chronons. To see whether my son is asleep takes me 100 chronons. Thus $d(\mathbf{E}_7, \mathbf{E}_{11}) = 100$ chronons. To take a plane and fly to New York takes me 5 kilochronons, which is 5000 chronons. Thus $d(\mathbf{E}_7, \mathbf{E}_{11}) = 5$ kilochronons.

The function d is not really a distance (in the mathematical sense). Indeed, the magnitude of $d(\mathbf{E}, \mathbf{F})$ not only depends on the experiences $\mathbf{E}$ and $\mathbf{F}$, but also on the way we take to switch from experience $\mathbf{E}$ to experience $\mathbf{F}$. Usually there are different ways. To get from "reading the novel" to "being in New York" can be done in different ways, by means of different intermediate experiences. The distance, if it is to be made objective (intersubjective), should be defined on the reality, hence on the collection of happenings. For two happenings $\mathbf{H}$ and $\mathbf{I}$ we can take $d(\mathbf{H}, \mathbf{I})$ to be the smallest (highest lower bound) of all the $d(\mathbf{E}, \mathbf{F})$ where $\mathbf{E}$ is an experience which contains the happening $\mathbf{H}$, and $\mathbf{F}$ is an experience which contains the happening $\mathbf{I}$. But also this is not yet a distance in the mathematical sense, because there also exist different ways to get from happening $\mathbf{H}$ to happening $\mathbf{I}$, even if we consider for every way the smallest one concerning the creation aspect. This is just the expression of the fact that my reality has more than one dimension (in a reality of one dimension only one sequence of intermediate happenings would exist between two arbitrary happenings).[8]

[8]I mention that the fact that reality has more than one dimension is directly related to its construction, and to the fact that we accept that at any moment I (and other humans, and animals, and plants, and . . .) have a free will to choose partly which creation I will fuse with which happening. A material particle that follows its path obeying Newton's deterministic laws (or the deterministic laws of general relativity) is in fact "existing" in a one-dimensional reality. Indeed, two arbitrary happenings of this material particle can only be connected by one intermediate sequence of happenings. More concretely, I live in a macroscopic world of more than one dimension because I can decide at any moment to move forward or backward,

These different ways will take in general a different amount of time, and hence lead to different values for $d(\mathbf{H}, \mathbf{I})$. We come to a real distance if there is a consensus about which road between two happenings is going to be taken as the representative one for the distance between these two happenings. Such a consensus has existed probably many times for the human species. I have often heard my grandfather tell me that in his time, the distances between different places (cities, towns, villages) were expressed in hours and minutes, and the consensus was the time needed for an average person to go (by feet) from one place to the other following a particular road. In modern times this consensus has lost its utility, because there are so many different ways of transportation and so many different roads for a person to go from one place to another.

If the skier has a watch, material time is defined for him by the number indicated on the watch. To ski from one spot of the hill to another spot of the hill can take him different amounts of time, depending on his speed, and on the road he takes. Also his reality, the shape of the mountain, has more than one dimension.

I give an example of a construction of a reality, to explain in more detail the present level of reasoning and to make clear that we are still far from the construction of material space.

Suppose we consider the situation where I am watching television, and this watching of television is the only type of experience that I can have. The only power of creation I have is to change the channel. Then it is clear that although my present experience consists of watching one particular channel, I make the construction, as explained in Section 8, and the other channels exist in my present reality as happenings, because I could have chosen to watch one of the other channels in replacement of the actual channel that I am watching. The topological structure of this kind of minireality is worth specifying. The switching from one channel to another one takes me the same amount of time, and since I only have one manner of switching, the distance is uniquely defined. It will be the same between all channels. I can represent this minireality by locating all the channels on the surface of a sphere with radius r, and myself in the center of this sphere. The distance between each channel is then equal to $2 \cdot r$. It is important to

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to the left or to the right, upward or downward. We humans also classify all the other material objects (living or not) in this higher-dimensional space because every one of us can take such a material object in his or her hands and "freely" choose to make it move in different directions (of course against the laws of Newton). In this aspect already the formalism that we present here differs from the ordinary physical theories. These physical theories try to make a model for the world and also for me, such that also I am controlled and created by the physical laws that control and create the material particles of the world. In our approach, the concept of (human) **will** is a primitive concept that we shall not try to deduce from other concepts.

remark that although this very poor situation is sufficient already to construct a reality, it does not give me any information about the distance between the different channels. But a representation (because we can easily imagine the real situation that we have on our actual televisions) needs already a reality of more than one dimension. I analyze this problem in much more detail in Aerts (1990*c*). Here I only mention it to prepare for the thoughts expressed in the next section.

12. THE FLOW OF REALITY

As follows immediately from our analysis, reality is not given once and for all. Other living beings, even on our own planet, must have constructed a completely different reality than our human reality.[9] Insects, of which some have certainly developed a high level of social life (e.g., bees and ants), have probably never started to study the periodic motions of the planets and stars. Of course part of our reality (the happenings) is certainly also valid for them, because of the simple fact that they are part of our reality. For example, also their bodies are made of matter, and this matter is the same one that we have studied and tried to understand in our human physical theories. But this part of our reality is probably no part of their reality, because they

[9]It would be very illuminating to study by means of the methodology that we have introduced in this paper the realities of other living beings. Parts of such studies have been undertaken. I mention, for example, the episode on the physics of vision recounted by Feynman (1970). Feynman compares the human eye with the insect eye, and indicates some aspects for the construction of reality. For example, a human eye is sensitive for light from red (7000 angstroms) to violet (4000 angstroms). A bee, however, cannot see red but can see ultraviolet (down to 3000 angstroms). Therefore bees can distinguish between many flowers which to us look alike. Apparently white is not very interesting to the bees, because all these white flowers have different proportions in the ultraviolet; they do not reflect 100% of the ultraviolet as would a true white. All the light is not coming back, the ultraviolet is missing, and that is a color for the bees, just as for us, when the blue is missing, the flower is yellow. So all the flowers are colored for the bees. What about the red flowers. Are they black for the bees? Not so! A careful study of red flowers shows, first, that even with our own eye we can see that a great majority of red flowers have a bluish tinge because they are mainly reflecting an additional amount in the blue, which is the part that the bee sees. Furthermore, experiments also show that flowers vary in their reflection of the ultraviolet. So if we could see the flowers as bees see them, they would be even more beautiful and varied. It has been shown, however, that there are a few red flowers which do not reflect in the blue or in the ultraviolet, and would therefore appear black to the bee. This was of quite some concern to the people who worry about this matter, because black does not seem like an interesting color, since it is hard to tell from a shadow. It actually turned out that these flowers are not visited by bees, but by hummingbirds, and hummingbirds can see the red.

cannot fuse their creations with these happenings. We cannot really imagine what their relation to matter is.[10]

It is rather obvious that these remarks are even more true for the entities that constitute matter. These entities are, for the simple fact that they constitute matter, **not matter**. But independently of this fact they seem to be entities. Exactly as a walnut before being cracked contains the happening $\mathbf{H_{13}}$**(the walnut is potentially eatable and potentially uneatable)**, these pre-material entities seem to contain the happening $\mathbf{H_{14}}$**(the entity is potentially localized and potentially nonlocalized in a region R of space)**. This actually means that these pre-material entities do not really exist inside space as we know it (exactly as a walnut is not yet cracked). The absorption and structuring of one of these pre-material entities in a matter-structure brings it into a state inside space as we know it, exactly as the cracking of the walnut brings it into a state of eatable or uneatable walnut.

To understand better this analogy of how structuring can bring happenings of the potential type into happenings of the actual type, we will give an example to show that the same process is still going on. Before nations existed, humans had no nationality. Let us consider such a concrete human being named Boduognat, and also consider all his descendants. Then because of the structuring process of society into nations, we know that all his descendants that live now have some kind of nationality. We can consider the happening $\mathbf{H_{15}}$**(this person has Belgian nationality)** and the inverse happening $\mathbf{H'_{15}}$**(this person has non-Belgian nationality)**. Then for each descendant of Boduognat, the happening $\mathbf{H_{16}}$**(this person has potentially Belgian nationality and potentially non-Belgian nationality)** has changed into one of the two happenings $\mathbf{H_{15}}$**(this person has Belgian nationality)** or $\mathbf{H'_{15}}$**(this person has non-Belgian nationality)**. That there once were humans without nationality (hence in our world such a human is neither a Belgian nor a non-Belgian, because every non-Belgian in our actual world has another nationality) is not difficult for us to imagine, and hence believe. This because we still consciously know the structuring process that led to the situation where all the humans of the world were given a nationality. To imagine that there are entities that have no place in space (are not localized) is more difficult for us. This is because we do not know consciously the structuring process that led to space and its macroscopic entities.

A human without nationality could be described as a superposition of different nationalities, namely those nationalities that he or she could take during the process of structuring. Let us call Ψ a certain descendant of

[10]Schrödinger (1944) studies in detail, relying on the second law of thermodynamics, why a kind of stable intelligent life could only have developed by collecting an amount of matter of the order of the amount of matter collected in the bodies of the living entities on earth.

Boduognat who will be nationalized in a certain country. Then we can write $\Psi = a \cdot \text{Belgian} + b \cdot \text{French} + c \cdot \text{German} + d \cdot \text{Dutch}$. The weights a, b, c, and d of each nationality must correspond to the probability that this person will be nationalized in this country.

In the same sense the wave function $\Psi(x, y, z)$ of a quantum mechanical entity has to be interpreted. It is a superposition of different points (x, y, z) of space, where the magnitude of the absolute value of the wave function in a certain point does not represent a "presence" at this point (as would be the case if the wave function is interpreted as a field), but is related to the probability that the quantum entity will localize at this point if the process of structuring of matter (the measurement corresponding to the detection) takes place.

It was my aim to point out that this state of affairs is not mysterious if we use analogies of similar situations and processes in our everyday world. Evidently many other examples can be given [see the last section of Aerts (1990*a*)].

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